

# MOMENTUM ANALYSIS OF TORNADO WIND ENERGY CONCENTRATOR SYSTEMS

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ABSTRACT

The momentum theory is applied to the tornado wind energy concentrator system. It is shown that the power coefficient of the system can be written as a product of three factors: a mass concentration coefficient, an energy augmentation coefficient and an extraction coefficient. By this splitting, the relative importance of the three components in the power output of the system can be analysed. On small scale systems, including a real turbine, the three coefficients are determined and inherent drawbacks of the tornado system are detected. Extrapolation of the characteristic coefficients for large scale systems is possible leading to a performance prediction for real applications.

KEYWORDS - Tornado wind energy system    Momentum theory    Mass concentration coefficient    Energy augmentation coefficient    Extraction coefficient

INTRODUCTION

The Tornado Wind Energy System (TWES) was introduced by Yen [1,2]. Figure 1 shows the concept. The main part of the system is a tower which collects the wind through adjustable vanes that are opened at the windward side and closed at the leeward side. A vortex is created in the tower with a low pressure core that sucks air through a turbine in the bottom of the tower.

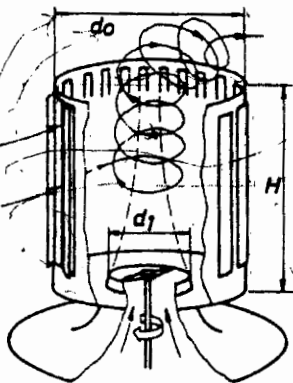


Fig. 1

Global performance predictions for the tornado system, based on moment considerations were formulated by Loth [3] and by Dick [4]. A general momentum theory for wind energy concentrator systems, separating the characteristics of the turbine and the concentrator parts was formulated by Dick [5]. In this paper, this general momentum theory is applied to the tornado wind energy system.

MOMENTUM THEORY FOR FREE TURBINES

In order to be able to apply momentum theory to a tornado system, it is necessary to recall the classic momentum theory of Betz for free turbines [6].

Figure 2 shows the flow through a wind turbine, schematically represented by an actuator disk. The undisturbed flow far upstream of the wind turbine has velocity \$v\_0\$. The wind velocity at the turbine is \$v\_1\$. Far downstream of the wind turbine, the velocity is \$v\_2\$. The air is assumed to be incompressible with density \$\rho\$. Due to the energy extraction by the turbine, the wind velocity \$v\_2\$ is smaller than \$v\_0\$. Hence, the streamtube through the turbine diverges. Some of the retardation of the flow takes place in front of the turbine, some behind the turbine. A uniform static pressure drop \$p^+ - p^-\$ over the turbine expresses the energy extraction. In the wake, the static pressure recovers to the free stream value \$p\_2 = p\_0\$. The pressure profile is sketched in figure 2.

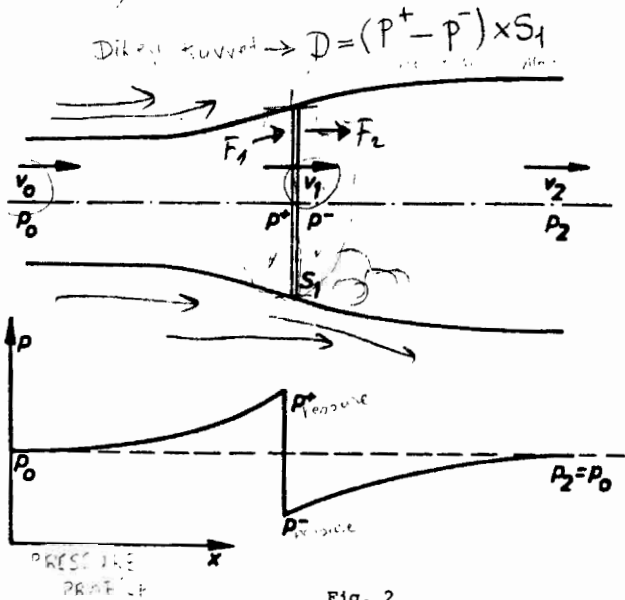


Fig. 2

From the law of conservation of momentum, it follows that the axial force  $D = (p^+ - p^-)S_1$  is equal to the decrease of the momentum flux over the turbine.

Assuming inviscid flow, there cannot be an exchange of enthalpy without mass transfer. Hence, the region influenced by the wind turbine is limited to the streamtube through the turbine. The decrease in momentum is thus:

$$\rho v_1 S_1 (v_0 - v_2) = D = (p^+ - p^-) S_1 \quad (1)$$

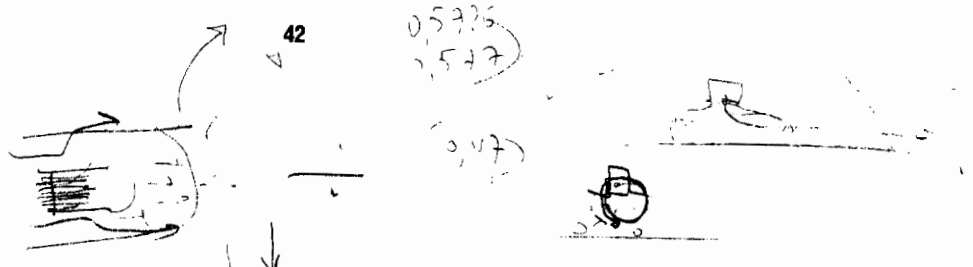
Since there is no enthalpy exchange upstream and downstream of the turbine, the Bernoulli equation holds for these parts of the streamtube:

$$p^+ + \frac{1}{2} \rho v_1^2 = p_0 + \frac{1}{2} \rho v_0^2$$

$$p^- + \frac{1}{2} \rho v_1^2 = p_2 + \frac{1}{2} \rho v_2^2$$

Since  $p_2 = p_0$ , this leads to:

$$p^+ - p^- = \frac{1}{2} \rho (v_0^2 - v_2^2) \quad (2)$$



The total enthalpy drop across the turbine is

$$\Delta H = \frac{p^+ - p^-}{\rho} = \frac{1}{2} (v_0^2 - v_2^2)$$

Hence, the power extracted by the turbine is

$$P = \rho v_1 S_1 \Delta H = \frac{1}{2} \rho v_1 S_1 (v_0^2 - v_2^2)$$

The power can be expressed by the power coefficient

POWER COEFFICIENT: 
$$C_p = \frac{P}{\frac{1}{2} \rho S_1 v_0^3} = \left(\frac{v_1}{v_0}\right) \left(1 - \left(\frac{v_2}{v_0}\right)^2\right) \quad (3)$$

Combination of (1) and (2) gives

$$v_1 = \frac{v_0 + v_2}{2} \quad \text{or} \quad \frac{v_2}{v_0} = 2 \frac{v_1}{v_0} - 1 \quad (4)$$

Hence, half the retardation of the flow takes place upstream and half the retardation takes place downstream of the turbine.

The power coefficient expresses the ratio of the power extracted by the turbine to the energy flux in a streamtube in undisturbed flow with a cross section equal to the turbine through flow surface  $S_1$ . Hence, the expression

$$\frac{v_1}{v_0} \left(1 - \left(\frac{v_2}{v_0}\right)^2\right) \quad (5)$$

can be seen as an extraction coefficient, indicating the part of the energy extracted from the flow which passes through the turbine in unloaded conditions.

With the relationship (4), the extraction factor (5) has a maximum for  $v_1/v_0 = 2/3$ , equal to

BETZ Limit:  $C_{p,max} = 0.593$  (BETZ LIMIT)

Clearly, due to the upper limit of the extraction factor (6) and the rather limited energy flux which can pass in unloaded conditions through a free turbine, the power of a free turbine is rather limited. The aim of concentrator systems is to alleviate the power limit of a wind energy system. Principally, this can be done in three ways, which usually are combined: augmenting the mass flow passing through the system, augmenting the specific energy content of the wind and augmenting the energy extraction factor.

In unloaded conditions, for a free turbine, the velocity at the turbine ( $v_1$ ) is equal to  $v_0$ . By augmentation of the mass flow through the system, it is to be understood that in unloaded conditions the velocity at the turbine is made to be larger than the free stream velocity. A mass concentration coefficient then can be defined by [5]:

CONCENTRATION COEFFICIENT:  $C_c = v_1/v_0 \quad (7)$

From (2), it is seen that for a free turbine, the maximum enthalpy drop is the kinetic energy content of the free stream (reached for  $v_2 = 0$ ):

$$\Delta H_m = v_0^2/2$$

It is said that an energy augmentation effect is achieved if this maximum enthalpy drop is made larger than the free stream kinetic energy. An energy augmentation coefficient is defined by [5]:

ENERGY AUGMENTATION COEFFICIENT:  $C_a = 2\Delta H_m/v_0^2 \quad (8)$

The absolute maximum power of the system is then:

$$P_{max} = \rho v_1 S_1 \Delta H_m$$

and an extraction coefficient can be defined by:

EXTRACTION COEFFICIENT:  $C_e = P/P_{max} \quad (9)$

The aim of concentrator systems: 43.

- 1) Augmenting the mass flow passing through the system:
- 2) Augmenting the specific energy content of the wind:
- 3) Augmenting the energy extraction factor:

Clearly, the power coefficient of the system is given by:

$$C_p = \frac{P}{\rho v_0 \frac{1}{2} v_0^2 S_1} = \frac{\rho v_1 2\Delta H_m}{\rho v_0 \frac{1}{2} v_0^2 S_1} \frac{P}{P_{max}} = C_c C_a C_e \quad \text{POWER COEFFICIENT (10)}$$

It is clear a priori that mass concentration is a matter of exerting forces perpendicular to the flow, i.e. forces which do not transfer energy to the flow. Obviously, in order to reach an energy augmentation effect, forces which execute work have to be exerted on the flow.

The result of the Betz theorem (6) cannot be interpreted as an absolute result, since it is obtained by a number of approximations. The theory is an example of a streamtube theory. This means that the energy extraction by the turbine is calculated based on momentum balances, on streamtubes, neglecting enthalpy exchange between flows in different streamtubes. This is clearly an inviscid approximation.

In loaded conditions, there is always a spontaneous energy transfer to the wake of the system. It arises from turbulent mixing with surrounding air which is not affected by the system itself. This turbulent mixing leads to the wake decay. For free turbines, the phenomenon of turbulent mixing was discussed by Hütter [7]. According to his measurements, the average axial velocity in the streamtube through the turbine can be sketched as in figure 3. The axial velocity at the turbine is higher than predicted by inviscid streamtube theory. Downstream of the turbine, the axial velocity reaches a value that is slightly lower than predicted.

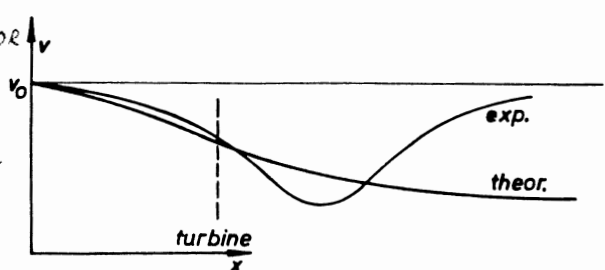


Fig. 3

Except for very low values of  $v_2$ , the functional relation between the minimal axial velocity  $v_2$  and the axial velocity at the turbine  $v_1$  can be approximated by

$$\frac{v_2}{v_0} = 2 \frac{v_1}{v_0} - 1 - 0.69 \left(\frac{v_1}{v_0} - 1\right)^2 \quad (11)$$

Neglecting the energy transfer to the wake due to turbulent mixing, before the point of minimum axial velocity, the power extracted by the turbine is still given by equation (3). With equation (11), the power coefficient reaches a maximum for

$$v_1/v_0 = 0.676 \quad v_2/v_0 = 0.280 \quad \text{POWER COEFFICIENT: } C_{p,max} = 0.623$$

leading to

$$C_{p,max} = 0.623 \quad \text{Calculation: } 0.676 \times [1 - (0.28)^2] = 0.623$$

This shows that, by the mixing of the wake and the surrounding flow, (an increase in extractable energy) may be expected in the order of 5%.

For low values of  $v_2/v_0$ , it is not possible to determine a clear functional relationship between  $v_2/v_0$  and  $v_1/v_0$  due to the breakdown of the regular flow pattern with the occurrence of a turbulent wake state.

In practice, the energy transfer to the wake due to turbulent mixing can be taken into account by not measuring  $v_2$  but by calculating it from the axial force coefficient of the turbine:

$$C_D = \frac{D}{\frac{1}{2} \rho v_0^2 S_1} = \frac{p^+ - p^-}{\frac{1}{2} \rho v_0^2} \quad \text{AXIAL FORCE COEFFICIENT}$$

From (2), it follows

$$C_D = 1 - \left(\frac{v_2}{v_0}\right)^2 \quad \text{SYSTEM FUNCTION} \quad (12)$$

Since the difference between  $v_2$  calculated by (12) and the actual  $v_2$  downstream of the turbine is limited to a few percent, values of  $C_D$  larger than 1 only occur in turbulent wake state, i.e. in load conditions which are far away from optimum. Hence by using (12), it is not necessary to incorporate the energy augmentation effect due to turbulent mixing into the energy augmentation coefficient. To do so would be very difficult since this effect is not determined by the system itself. Therefore, in the sequel,  $v_2$  always will be defined by (12). The functional relationship between  $v_2$  and  $v_1$ , obtained in this way, will be called the system function.

**MOMENTUM THEORY FOR THE TORNADO WIND ENERGY SYSTEM**

A lot of studies, both analytically and experimentally, were done on the vortex structure in the tornado tower. A thorough analytical study can be found in [8], an experimental study on towers with closed bottom walls in [9], experimental studies on towers in which the turbine is replaced by a screen in [10] and [11] and an experimental study with a real turbine in [12]. All these studies agree that the vortex in the tower can be seen as composed of a core region ( $0 < r < r_c$ ) with approximately solid body rotation and an outer region ( $r_c < r < r_0$ ) with approximately potential vortex structure. The tangential velocity profile can be well represented by a Rankine-law:

RANKINE LAW 
$$w = (K/r)(1 - \exp(-Re r^2/2 r_0^2)) \quad (13)$$

in which  $v_0$  is the free stream velocity and  $Re$  is a Reynolds number. This structure is asymptotically limited by an ideal solid body rotation in the core:

$$w = w_m (r/r_c) \quad (14)$$

and an ideal potential vortex in the outer region:

$$w = K/r \quad \text{with} \quad w_m = K/r_c \quad (15)$$

Experimentally, the tangential velocity at  $r = r_0$  is found to be slightly lower than the free stream velocity  $v_0$ . Therefore, it is believed that a theoretical upper limit of  $K$  is  $v_0 r_0$ . In the core region, pressure is approximately constant. In the outer region of the vortex, centrifugal force and pressure gradient are in equilibrium [9]:

$$\frac{1}{\rho} \frac{dp}{dr} = \frac{w^2}{r} \quad (16)$$

For  $r = r_0$ , pressure is approximately equal to the free stream pressure  $p_0$ . Measured tangential velocity - and pressure profiles are shown in figure 4E [10].

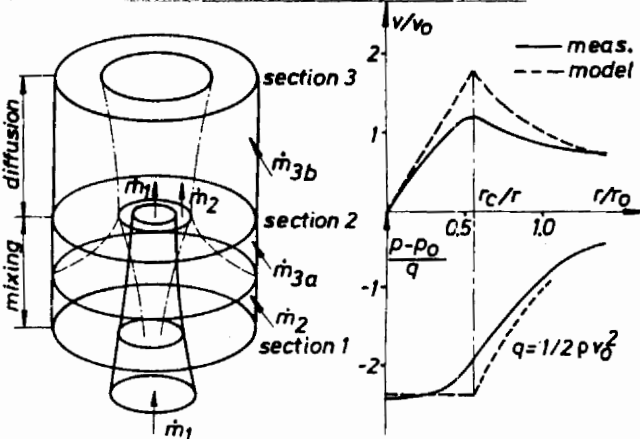


Fig. 4

According to the measurements described above, the tangential velocity profile can be modelled by (14) (15) with  $K = v_0 r_0$ . With the potential vortex structure (15), the pressure in the outer region follows from (16):

$$\frac{1}{\rho} (p_0 - p) = \frac{v_0^2}{2} \left( \left(\frac{r_0}{r}\right)^2 - 1 \right) \quad (17)$$

The result (17) implies that the total enthalpy in the potential region of the vortex is equal to the free stream value. This shows that there is no energy exchange between the flow in the potential region and the turbine and that the measured decrease in total enthalpy with decreasing radius in the potential region is due to losses. As a consequence, only the core region plays a role in the energy exchange between flow and turbine.

The complete flow pattern is sketched in figure 4a. In the lower part of the tower the Rankine-like shape of the tangential velocity profile is not yet established due to the mixing of the turbine flow and some of the flow that enters through the vanes. The lowest horizontal cross section in which the mixing is completed, i.e. in which the Rankine-law can be detected, is called the mixed-out section (section 2). The turbine flow is called the primary flow. The flow through the vanes which is collected in the core region of section 2 is called the secondary flow. The flow which comes in the potential region of section 2 and which enters in higher parts of the tower, is called the tertiary flow.

The ratio of secondary to primary mass flow can be determined theoretically by expressing conservation of moment of momentum between section 1 and section 2. In case the turbine is replaced by a screen, the moment of momentum just behind the turbine can be neglected and conservation of moment of momentum, based on the model flow, gives:

$$\int_0^{r_c} \rho v_2 w r^2 dr + \int_{r_c}^R \rho v K^2 r dr = K(\dot{m}_2 + \dot{m}_3), \quad K = v_0 r_0$$

The axial velocity in the core region in section (2) is denoted by  $v_2$  while  $v$  is the axial velocity in the potential region. By assuming  $v_2$  to be constant:

$$\frac{1}{2} K(\dot{m}_1 + \dot{m}_2) + K\dot{m}_3 = K(\dot{m}_2 + \dot{m}_3) \quad \text{or} \quad \dot{m}_2/\dot{m}_1 = 1 \quad (18)$$

The result (18) only holds for the model flow. In a real system, the secondary mass flow is lower than the value predicted by (18).

It is to be remarked that the mass flow ratio can be influenced by the moment of momentum in the primary flow. By denoting by  $K_T \dot{m}_1$  the flux of moment of momentum in the turbine flow, the balance (18) becomes:

$$\frac{1}{2} K(\dot{m}_1 + \dot{m}_2) + K\dot{m}_3 = K(\dot{m}_2 + \dot{m}_3) + K_T \dot{m}_1 \quad \text{or} \quad \frac{\dot{m}_2}{\dot{m}_1} = 1 - \frac{2K_T}{K} \quad (19)$$

Clearly, by giving a postrotation in the turbine flow, opposite to the sense of rotation of the vortex in the tower, the secondary mass flow can be increased.

Due to the mixing between secondary and primary flow, an energy transfer between both flows is possible. By assuming that this energy transfer occurs without losses, an upper limit of the power of the turbine can be found from an energy balance on the core flow between section 2 and a free stream position for both the primary and secondary flow [4].

From measurements [10] it is clear that the pressure which is maintained in the mixed-out section, in loaded conditions of the turbine, is below ambient pressure by an amount which is much higher than the dynamic pressure in this section. This means that the core flow has to recover to ambient pressure while energy is transferred from the tertiary flow by mixing. It is clear that this combined diffusion-mixing process actually determines the pressure that can be maintained in the mixed-out section. By assuming that the energy transfer from the tertiary flow occurs without losses, an upper limit for the power of the turbine can be found from an energy balance between section (3) and free stream positions for all flows involved [3].

The distinction between secondary and tertiary mass flows is very fundamental in the understanding of the energy transfer processes in the tornado system. Detailed measurements by Windrich and Fricke [11] show that the mixed-out section is an equilibrium section without radial flow and without axial pressure gradient. A consequence of this observation is that the flow field in the mixed-out section is two-dimensional, such that the tangential velocity profile and the pressure profile in the mixed-out section are independent of turbine load and tower exit conditions. It is clear that this only can be valid, if the velocity and pressure field in the mixed-out section can freely develop. This implies a sufficient height of the tower. This was clearly demonstrated in [10] in which towers with free suction are used in contrast to the towers in [11] in which by means of a blower the pressure in front of the screen is kept at atmospheric pressure, independent of turbine load. In [10] it was shown that the diffusion process between mixed-out section and tower top requires a minimum axial length in order to avoid return flow. For aspect ratios  $H/d_0$  higher than 2.5, a regular diffusion, involving only positive axial velocities is observed, leading to a well developed mixed-out section with a pressure profile that is two-dimensionally determined and is independent of tower height. As a consequence, for tower aspect ratios higher than 2.5, the power that can be extracted by the turbine becomes independent of tower height. For aspect ratios lower than 2.5 return flow in the axial diffusion is observed leading to a decrease of the subpression in the core of the mixed-out section and a decrease in extractable power. As a consequence, based on frontal area, the power coefficient reaches a maximum for the aspect ratio 2.5, i.e. at the onset of return flow, independent of other system parameters.

The existence of a mixed-out section, observed experimentally, can be used to simplify the theoretical analysis of the tornado system. Since, at least for sufficiently high aspect ratios, the pressure profile which is maintained in the mixed-out section is two-dimensionally determined, i.e. only dependent on the form of the cross section of the tower and its diameter, independent of load, the turbine flow can be considered to exhaust in a constant pressure reservoir similar to an infinite atmosphere. This makes it possible to extend classical momentum theory for free turbines to tornado concentrators as was principally shown in [5]. In this section, the momentum analysis of the tornado concentrator is further developed based on the concepts that were introduced in [5] for general concentrator systems. In contrast to [3] and [4], in the sequel, momentum theory is used in the classical sense of Betz. This means that the energy balance is taken on the streamtube passing through the turbine between mixed-out section 2 and free stream conditions, neglecting energy transfer due to mixing. A correction for mixing can be introduced, as shown in the previous section, by calculating  $v_2$  from the axial force coefficient of the turbine by (12).

When energy transfer in the streamtube through the turbine is neglected upstream of the turbine and between the free and mixed-out section, the Bernoulli equation holds for these parts of the streamtube :

$$p^+ + \frac{1}{2} \rho v_1^2 = p_0 + \frac{1}{2} \rho v_0^2 \quad p^- + \frac{1}{2} \rho v_1^2 = p_2 + \frac{1}{2} \rho v_2^2$$

$$\text{Hence : } p^+ - p^- = p_0 + \frac{1}{2} \rho v_0^2 - \frac{1}{2} \rho v_2^2 - p_2 \quad (20)$$

In contrast to the free turbine  $p_2 \neq p_0$ , and as a consequence, in unloaded conditions,  $v_2$  is different from  $v_0$  :

$$\frac{1}{2} \rho v_2^2 = p_0 + \frac{1}{2} \rho v_0^2 - p_2 \quad (21)$$

Hence, according to the definition (8), the energy augmentation coefficient is :

$$C_a = v_{20}^2 / v_0^2 \quad (22)$$

Using (20) and (21), the power extracted by the turbine is given by :

$$P = \rho v_1 S_1 \frac{1}{2} (v_{20}^2 - v_0^2)$$

The power coefficient is :

$$C_p = \frac{P}{\frac{1}{2} \rho v_0^3 S_1} = \frac{v_1}{v_0} \left[ \left( \frac{v_{20}}{v_0} \right)^2 - \left( \frac{v_0}{v_0} \right)^2 \right] \quad (23)$$

Using the energy augmentation (22) and the mass concentration coefficient (7), (23) becomes :

$$C_p = \left( \frac{v_{10}}{v_0} \right) \left( \frac{v_{20}}{v_0} \right)^2 \left( \frac{v_1}{v_{10}} \right) \left[ 1 - \left( \frac{v_0}{v_{20}} \right)^2 \right] \quad (24)$$

Hence, the extraction coefficient has the form :

$$C_e = \frac{v_1}{v_{10}} \left[ 1 - \left( \frac{v_0}{v_{20}} \right)^2 \right] \quad (25)$$

It is clear that (25) is a generalisation of (5), since for the free turbine  $v_{10} = v_0$  and  $v_{20} = v_0$ .

From (20), it is seen that  $v_2/v_{20}$  can be calculated from the measured force coefficient of the turbine :

$$C_D = \frac{p^+ - p^-}{\frac{1}{2} \rho v_0^2} = C_a \left[ 1 - \left( \frac{v_0}{v_{20}} \right)^2 \right] \quad (26)$$

(26) is to be considered as a generalisation of (12).

#### EXPERIMENTAL VERIFICATION

It is clear that the quality of the vortex structure is strongly dependent on the geometry of the vanes. In order to circumvent the problem of designing optimal vanes, for analysis purposes a logarithmic spiral shaped tower was introduced by Yen [1,2] in which the ideal vortex structure (13,14) is easily reached. Results for models of this type with closed bottom walls were reported by Windrich et al. [9]. The same logarithmic spiral shape was used by Haers and Dick [10] in studies on towers with a bottom opening in which the turbine is simulated by a screen. A similar study was done by Windrich and Fricke [11]. In [12] also results are reported on cylindrical towers with vanes.

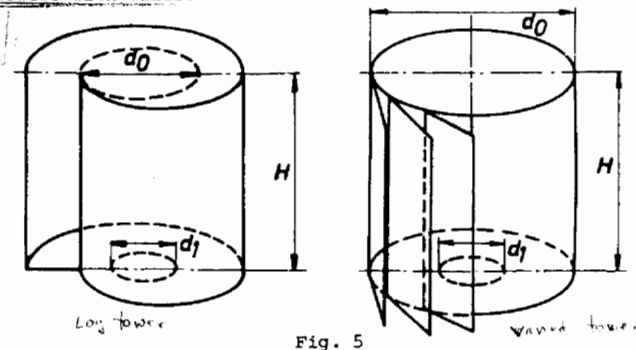


Figure 5 shows the logarithmic spiral tower used in [9-12]. The form of the spiral is :

$$r = r_0 e^{.1 \theta} \quad 0 \leq \theta \leq 2\pi$$

The logarithmic spiral tower is to be seen as a tornado tower with diameter  $d_0$  and one, large, vane collecting the wind in an optimal way. Therefore, the frontal area of the system is calculated as  $H d_0$ . Figure 5 also shows the model of the cylindrical tower used in [12]. Vanes are opened in one quarter of the tower.

Figure 6 shows the power coefficient, based on frontal area for the logarithmic tower and the cylindrical tower for several values of tower aspect ratio  $H/d_0$  and turbine diameter to tower diameter ratio  $d_1/d_0$ , in function of screen porosity  $\alpha$ . This figure shows that the performance of the omni-directional tower is consistently lower than the performance of the uni-directional

(logarithmic) tower. This is due to the lower quality of the vortex structure in the omni-directional tower. The tangential velocity profiles in the outer region of the vortex are asymptotically tangent to  $v = k v_0 r_0 / r$ .  $k$  is found to be approximately .8 for the uni-directional towers and .6 for the omni-directional towers.

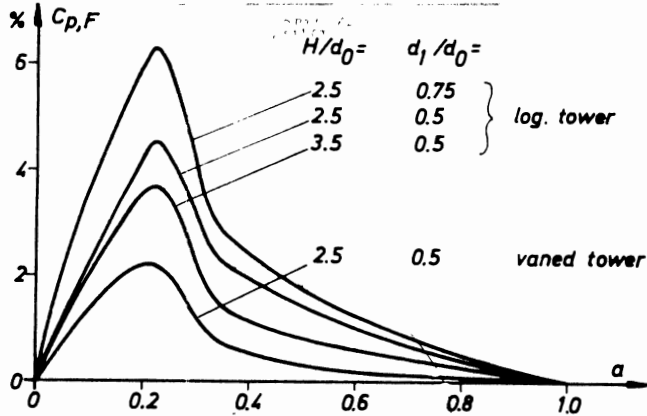


Fig. 6

For both tower forms, it was found that, with free suction, the optimal aspect ratio  $H/d_0$  is about 2.5. The optimal turbine diameter was found to coincide with the diameter of the core in the mixed-out section. For the small scale models used in [10-12] the optimal turbine diameter to tower diameter ratio was found to be about 0.5 to 0.7. It is however clear that for large systems this value should be much lower since according to (13) the core radius is inversely proportional to the Reynoldsnumber. It is easy to interpret the experimental results in terms of the momentum theory. From the mass flow in unloaded conditions,  $C_c$  can be calculated according to (7). From the pressure in the mixed-out section  $C_a$  can be calculated according to (21) and (22). The system function then can be obtained from the measured power coefficient and the measured mass flow. For the logarithmic spiral tower with  $H/d_0 = 2.5$  and  $d_1/d_0 = 0.5$ :  $C_c = 0.549$  and  $C_a = 2.143$ . For the omni-directional tower with  $H/d_0 = 2.5$  and  $d_1/d_0 = 0.5$ :  $C_c = 0.348$  and  $C_a = 1.538$ . The experimentally obtained system functions are shown in figure 7.

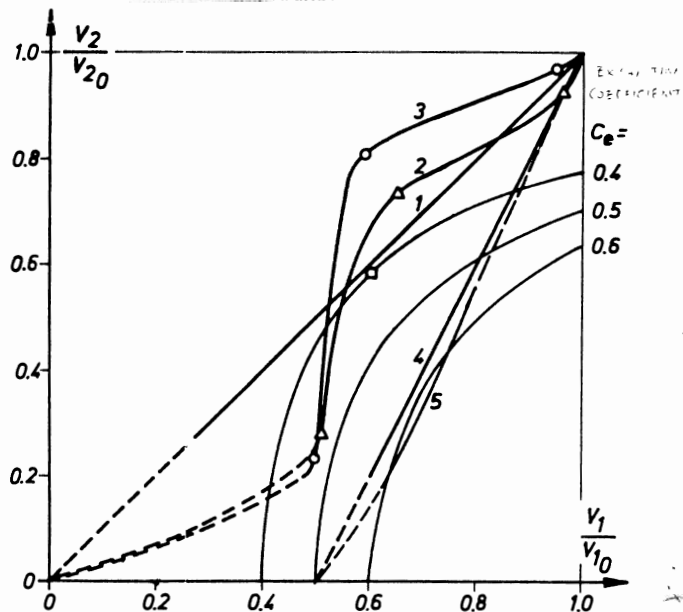


Fig. 7. System diagram

- 1: Theoretical system function of tornado system.
- 2: Experimental system function of log. tower (screens).
- 3: Experimental system function of vaned tower (screens).
- Logarithmic tower with real turbine.
- 4: Theoretical system function of free turbine.
- 5: Experimental system function of free turbine.

With the simplifying assumptions introduced above and for a constant mass flow ratio  $\dot{m}_2/\dot{m}_1$ , obviously there is a continuity relation between the flow passing through the turbine and the flow passing through the constant radius core of the mixed-out section such that the system function theoretically is:  $v_1/v_{10} = v_2/v_{20}$ .

The accordance between the experimental and theoretical system functions is reasonably good. It leads to the conclusion that there is approximately a balance between the neglected losses and the neglected energy transfer from the secondary to primary flow. For reasons of comparison, the theoretical and experimental system functions for a free turbine resulting from the Betz analysis (4) (11) are also shown in figure 7, together with curves of constant extraction factor. The theoretical maximum extraction coefficient for the free turbine is .593 (16/27), while for the tornado system it is only .385 (2/3√3). The theoretical optimum for the tornado system is obtained for

$$v_1/v_{10} = v_2/v_{20} = 1/\sqrt{3} = 0.577 \quad (27)$$

The experimentally determined optimum corresponds to:  $v_1/v_{10} = 0.5$   $v_2/v_{20} = 0.25$   $C_{e,max} = 0.47$  (28)

Figure 7 also shows the results of a tornado system with a real turbine. The turbine was not designed to match the experimentally obtained optimal conditions (28) since this would result in a high disk loading with corresponding blade forms similar to these in gas- and steam turbines. It was felt that the miniature construction of such a blade was too difficult. Therefore, load conditions corresponding to the theoretical optimum (27) were chosen. It was found that for these conditions, a turbine with approximately zero postrotation and approximately constant axial flow velocity could be designed based on NACA 4412 aerofoil sections with a blade torsion of 15° [12]. The point shown in figure 7 is the gross power point (power taken from the flow). The net power corresponds to 0.75 of the gross power. It is seen that the extraction coefficient of the real turbine is somewhat better than that of an equivalent screen. In [12] it was shown that the same performance is reached, independent of the postrotation in the flow.

#### CONCLUSION

It is clear that the performance of a large scale system can be much better than the performance measured with the small scale system used in this study to illustrate the momentum theory. The mass concentration coefficient and the energy augmentation coefficient can be expected to be much better for large scale systems. It is clear however that the product of these coefficients never can be larger than the frontal area to turbine area ratio, since mass concentration and energy augmentation express the transformation of the kinetic energy flux of the wind captured by the frontal area to the energy flux in the turbine flow. This means that based on frontal area, the power coefficient cannot be larger than the extraction coefficient. The momentum interpretation of the results for small scale systems shows that the system function of a tornado wind energy system is not as favourable as the system function of a free turbine. As a consequence it is to be expected that for large scale systems the gross power coefficient is lower than the Betz limit. It can be expected to be  $\approx 0.4$ .

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